THE OPTIMAL LOCATION OF EMERGENCY SERVICE TELEPHONES IN CENTRAL PARK

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ABSTRACT: Three related covering models, the Location Set Covering Problem (LSCP), the Maximal Covering Location Problem (MCLP), and the P-median problem (PMP) are discussed with regard to the optimal location of emergency service telephones in New York City's Central Park. Questions considered include: Are the current locations of phones optimal? Is the number of phones currently in the park sufficient for coverage? How different are the alternative optima for phone placement over a range of suitable parameters? The appropriateness and usefulness of each model discussed is critiqued. Finally, the necessary steps to develop a GIS to answer what-if questions over a range of model parameters are outlined.

The central theme of this paper is to define the location-allocation covering model, or models, best suited to the task of optimally locating emergency service telephones in Central Park -a unique and circumscribed emergency service precinct, offset from other precincts, with its own telephone system. The park is an extensive police precinct in the center of the most heavily trafficked borough of the primate city in the United States. Because of its function as a recreation area, access into and out of the park is limited. Anyone needing emergency service in the park, including police and ambulance service, must either find a phone on the parks independent network, or travel a long way out of the park to call for service. Currently 47 telephones comprise the parks independent telephone network. A factor that differentiates the park precinct from its neighbors is its extensive and open area. This must both complicate emergency response within the park, and make indispensable the proper allocation and use of the emergency phone network.

THE MODELS

Since Varignon's Frame, location modelers have attempted to find and quantify optimal solutions to the combinatorial problems they study. With the formulation of the Location Set Covering Problem (Toregas, Swain, ReVelle, and Bergman 1971), we gained the ability to incorporate service coverage into these models. The LSCP is formulated as follows:

LSCP

$$Min\ Z = \sum_j X_j$$

subject to:

$$\sum_{j \in N_i} X_j \geq 1 \qquad \forall i$$

$$X_j \in (0,1) \quad \forall j$$

where

I = the set of demand points
J = the set of potential facility sites
d_{ij} = the shortest distance from i to j
$N_i = \{j \in J \mid d_{ij} \leq S_i\}$
S_i = the maximal service distance for demand i
X_j = {1 if a facility is located at j
{0 if not

The LSCP finds the minimum number of service centers required to meet all demand such that there remains no demand outside a given service distance, S. The objective function asks for the least number of facilities. The first constraint specifies that for each demand point there must exist one facility within the specified distance. The second constraint restricts values of Xj to a zero-one integer to mitigate against partial assignment. Inherent to the problem formulation is the idea of a critical distance. It assumes a monotonically nonincreasing benefit curve, and seeks to find the least number of service centers such that all demand falls within the area of the curve. This notion jibes well with the needs of this paper's stated problem and the requirements of effective emergency response in general. The following example, specific to this study, will illustrate the idea: For a stroller in the park overcome by an heart attack, the existence of 47 or 447 telephones within the park is meaningless, unless a witness or friend can get to one within some critical distance, either Euclidian or temporal.

Another covering problem, the Maximal Covering Location Problem, is an important derivative of the LSCP (Church and ReVelle 1974). The key feature of the MCLP is that it maximizes coverage within a given critical distance, S. The objective function can be formulated either as a minimization or maximization problem. Formulated in the former manner, the model finds the solution that minimizes demand not covered within the given distance, S. Formulated in the latter manner, the model finds the solution that maximizes demand covered within S. Below the problem is stated in its minimization form as given in Church and Weaver (Church and Weaver 1986).

MCLP

$$Min \sum_i A_i Y_i$$

subject to:

$$\sum_{j \in N_i} X_j + Y_i \geq 1 \quad \forall i$$

$$\sum_j X_j = P \quad \forall i$$

$$X_j \, , \, Y_i \in (0,1) \quad \forall i \, , \, \forall j$$

where

A_i = the population served at demand point i
P = the number of facilities to be located
Y_i = {1 if demand i is *not* covered within S_i
{0 otherwise

and where all other notation is as before. The objective function seeks the minimum combination of population at uncovered demand points. The first constraint determines if a demand point is covered, that is, either at least one in the set of facility sites is within S of i, and therefore the sum of X_j is greater than or equal to one, or there are no facility sites within S of i, and Y_i equals one, for each i. The second constraint stipulates the number of facility sites to be located. The third constraint restricts the variables X_j and Y_i to zero or one to guard against partial assignment. Note that in the MCLP the number of facilities is intrinsic to the model, while this is not the case with the LSCP. The benefit curve of the MCLP is, like that of the LSCP, monotonically nonincreasing (Church and Roberts 1983). It finds the solution that minimizes the number of people or demand points that do not fall under the benefit curve. An example of the use of the MCLP follows: A decision maker who wishes to cover demand within some critical distance S, but does not have enough funding to supply the requisite facility sites, will nevertheless wish to maximize what coverage he is able to supply within that distance; for this the MCLP is used.

A third facility location model, the P-Median Problem, is not concerned with the notion of critical distance as outlined above. The P-Median Problem finds the unique solution that minimizes the average distance of all demand points to facility sites (ReVelle and Swain 1970). The goal of the P-Median Problem has been described as system wide efficiency (Ghosh and Rushton 1987). It is formulated as follows:

PMP

$$Min \sum_{i=1}^{n} \sum_{j=1}^{n} A_i D_{ij} X_{ij}$$

subject to:

$$\sum_{j=1}^{n} X_{ij} = 1 \quad \forall i \in I$$

$$\sum_{j=1}^{n} X_{ij} = P$$

$$X_{ij} \leq X_{jj} \qquad \forall i, \forall j, \quad i \neq j$$

$$X_{ij} \in (0,1)$$

where

X_{ij} = {1 if i assigns to facility j as its closest facility
{0 otherwise

n = the number of demand nodes

and all other notation is as before. The objective function finds the solution that minimizes the distance the population at demand point i will need to travel to get to facility site j, on average. The first constraint restricts assignment of each demand point i to only one facility site. The second constraint specifies the number of facility sites to be located. The third constraint insures that any site with a facility self-assigns. The fourth constraint restricts the values of Xij to zero or one. An example of the use of the PMP follows: A decision maker has enough funding to provide for a set amount of facility sightings, and, not concerned with any critical distance, wishes to minimize the average distance from the demand points to the facility points: for this the PMP is used. Specifically with regard to the concern of this paper, the PMP solves so that any place in the park will be a minimum distance from a phone, on average.

DISCUSSION

Before proceeding with a discussion of model choice, several attributes of the models, both positive and negative, need be noted:

1) The LSCP does not solve with a unique optimum, but solves only one of several alternate optima (Church and ReVelle 1974);

2) The MCLP has a special case, known as the MCLP with mandatory closeness constraints, which solves for the alternate optima of the LSCP (ibid);

3) Data aggregation, of continuous demand into point demand,causes errors in the solutions of these models that increases in a systematic way, as aggregation increases (Casillas 1987);

4) The center of a graph always falls on a vertex (Hakimi 1963);

5) The PMP, despite overall efficiency, may solve so that individual accessibility varies widely within the system (Ghosh and Rushton);

6) The PMP assumes the infinite capacity of each facility to meet demand (Church and Weaver 1986);

7) The PMP assumes that each demand node always assigns to its closest facility -analogous to the economists notion of perfect rationality (ibid).

The first property should not cause great concern. Though there exist alternate optima, at least we know we have found one. If it does worry us overly, we can take advantage of the second property and solve the MCLP with mandatory closeness constraints for a range of values as was advised by its authors (Church and ReVelle 1974). The third property is the most alarming. There is little, as far as this writer knows, that can meliorate the effects of data aggregation or preclude the subsequent errors. We can attempt to minimize error propagation by keeping the level of aggregation uniform over the study area. Furthermore, a GIS can help us in this area by allowing us to trade higher levels of aggregation for the computer's computation efficiency. The fourth property should simply make us cheer.

Having identified the differing assumptions of three classic covering models, we are left with the following dilemma: Which one should we use? The short answer is, of course, that it depends. The answer given here, however, is that the model which best matches the goals of the decision maker is the one to use. We should keep in mind when deciding upon which to use, the role of the different variables in each. In the LSCP, the maximum, or critical distance is given, and the model determines the minimum set of facility sites that covers all demand points within it. In the MCLP, the critical distance is again given, but so is the number of facility sites. The model identifies the maximum set of demand points that are covered, or, alternatively, the minimum set not covered. In the P-Median problem, the number of facilities to be located is given, but no distance threshold is specified. The model determines the set of facility sites that minimize the average distance of demand points to facility points.

At this point, the problem of model choice should seem trivial. We have a clear understanding of what each model does, and need only pick the one which best fits our intentions. We can run a type of location-allocation sensitivity analysis by solving the LSCP for a range of values of S and graphing the results. This was suggested by Toregas and ReVelle (Toregas and ReVelle 1972). This operation would yield a trade-off curve between levels of funding and distance of coverage. If the critical distance is known, and just sufficient funding exists to locate the minimum facilities to cover all demand within that distance, we will use the LSCP. The information on combinations of minimum facility numbers and funding levels would be gleaned directly from the trade-off curve we generated. If insufficient funding is available, we will probably like to still do the best with what is available, and so will use the MCLP to maximize critical coverage. If more funding is available than is required to locate just the minimum to totally cover demand within the critical distance, we will use the PMP, for we will have the luxury of maximizing system efficiency while still preserving coverage of demand within the critical threshold.

Actually, the model preferred by the author is the PMP. The PMP is the only one of the three models for which we can manipulate values of the distance variable D_{ij} in a meaningful way. In the LSCP or MCLP, we can weight the distance variable, but it is not contained within the objective function. This means, essentially, that weighting any of the distances in the LSCP or MCLP would be tantamount to relocating the weighted point in space. Weighting of distances in the PMP can be accomplished by attaching a weighted variable, say W_{ij}, to the distance variable for each facility site. The effect of such an addition to the model is to increase the importance of some facility sights over others and some facility-site demand point relationships over others. This is because the PMP treats the distance variable in the objective function. Weighting the distance in the LSCP or MCLP could put the demand point outside the critical range of any facility site, or, alternately, inside it, in an artificial way whether the demand point or potential facility site is weighted. Weighting the distance in the PMP would cause the demand to either be pulled toward the facility, or relax the need for proximity. Candidate facility points for positive weighting include sites at the entrances to the park and sites along main thoroughfares where pedestrians might expect and look for emergency phones.

INTEGRATING A GIS

The need for a GIS to aid in the completion of this project is real and great. Calculating the distance variable, D_{ij}, from our potential facility sites to our aggregated demand points alone warrants incorporating a GIS. But the utility of a GIS to a study such as ours goes much farther than this simple, though beneficial, operation. With a GIS we will be able to solve the LSCP for a range of values, S, and generate our trade-off curve between funding and coverage. With a GIS we will be able to quickly and easily identify special case facility points, like park entrances and exits and the circumferential road inside the park, Park drive. We will be able to test the model, if we use the PMP, for a range of weights for these special case demand points. Furthermore, any physical changes made to the park, whether landscape changes or changes in the number of phones, can be quickly and easily incorporated or updated into the GIS. The GIS should be a vector based system. A fine mesh Cartesian grid overlaid on the vectors that define both the paths in the park and the polygons that represent park open areas will be used to calculate distances from aggregated demand points to potential facility sites and the set of potential facility sites at vertices on the grid. In this manner we can test the effects of aggregation on our system, for we can alter the fineness of the grid. The computational abilities of the computer together with the flexibility for testing hypothetical situations makes GIS integral to the elegant solution of this paper's problem, optimizing the location of emergency service telephones in New York City's Central Park.

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