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# MEREOTOPOLOGICAL INTEGRITY CONSTRAINTS FOR SPATIAL DATABASES

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***ABSTRACT:*** *This paper suggests a framework for defining a new class of the integrity constraints for spatial databases that employ 'part-of' relations between the types of objects. Other relations such as connectedness can be deduced automatically using a small number of axioms. The study draws its formalisms from such areas of philosophical ontology as mereology, mereotopology and theory of granular partitions. Mereotopological constraints can be applied to object-relational and object-oriented spatial databases.*

## INTRODUCTION

Databases store information on some part of the real world. There are always many rules that govern relationships between the entities of the real world. In databases such rules are represented as integrity constraints. Integrity constraints can be implicit for database schemata or inherent to the data model itself. For example, in the relational database model a uniqueness of keys, entity integrity and referential integrity between tables must be obeyed for the proper functioning of a database system (Elmasri, 2000). However, many rules that exist in the real world do not have their counterparts in the data models and have to be specified explicitly. Examples of such constraints are the facts that the age of a person must be in the range between 0 and 120 years and that a salary of an employee must not be negative. The importance of constraints grows with the currently observed increase in the size, diversity and interconnectivity of the databases. As a result new automated methods of ensuring database integrity have to be introduced.

It is possible to infer a large number of the constraints using relatively few basic rules (Brodsky, 1997). The task of specifying integrity constraints can be simplified if the knowledge of common structures underlying our understanding of the reality is used. Spatial databases have an advantage of employing properties of space as integrity constraints such as the ones that are imposed by the topological vector data model (Laurini and Thompson, 1992). However there are many other relations in the geographic reality that have a potential to be used to enforce integrity of the databases. This paper tries to outline the conditions to employ the "part-of" relationship between geographic objects for this purpose.

## SPATIAL INTEGRITY CONSTRAINTS

As it was already mentioned, many of the constraints in spatial databases can be based on the underlying properties of space. Cockcroft (1997) subdivides spatial integrity constraints into three types: (1) topological, (2) semantic and (3) user-defined.

*Topological constraints* are based on the topological properties of the metric space. The topological vector data model that has been commonly used for several decades offers a set of simple constraints that can be applied to a significant number of cartographic representations. The vector topological model prohibits overlapping surfaces, disconnected edges or edges that subdivide regions with the same categorical value.

*Semantic constraints* are ones that can be derived from the properties of geographic objects.

For example, a road and a river cannot coincide unless there is a bridge.

*User-defined constraints* represent rules that are arbitrarily imposed upon the database. For example, a residential house should be no further than a certain distance from a fire hydrant.

Introduction and wide acceptance of object-oriented spatial databases in the 1990s has brought new challenges to the area of spatial integrity. Flexibility of the object-oriented model allows the definition of an unlimited number of relationships between the objects. However, one of the major problems of object-oriented databases is the lack of inherent integrity constraints that forces developers to specify many explicit constraints (for examples see Borges et al., 1999). Topological constraints can be successfully used in an object-oriented model, too (Belussi et al., 2000). In this study we are trying to define new types of constraints that are rooted in the part-whole relation and would be as general as topological constraints and could be applied to a variety of underlying data models.

## ONTOLOGICAL FOUNDATIONS

Geographic objects do not occur in space on their own but rather they form various systems. One of the ways to organize objects of the real world is to create hierarchies that align objects along the "part-of" relationship (Mark et al., 1999). The part-whole relation is very easy to understand on the level of intuition and there is a plethora of "part-of" relation examples both in everyday life and in science: a hand is a "part-of" the body, a mountain is a "part-of" a mountain range, a slope is comprised of upper and middle parts and a toe. Properties of such hierarchies are studied in the field of ontology called "mereology". Mereotopology extends mereology to handle the notions of continuity, boundedness and connectedness. Mereology is a very well studied and formalized area of philosophical ontology and has been under investigation for at least one hundred years. Good formalization and abundance of part-whole relationships in the geographic world create a potential to employ mereological and mereotopological relations to specify integrity constraints for spatial databases.

The theoretical foundation for this study is comprised of three theories: (1) a theory of parts and wholes that in philosophical literature is referred as mereology, (2) mereotopology that extends mereology with the relation of connectedness and (3) a theory of granular partitions that provides mechanisms for the building of classifications. Formalization of the theory will be presented using first-order predicate logic with variables designated with letters $x, y, z, a, b, b_1, \ldots$ and classes as $X, Y, Z, A, B, B_1, \ldots$. Mostly we will use commonly accepted symbols. Symbols that are not so common will be explained in due course. Each formal statement will be provided with a verbal explanation and then summarized in an informal discussion.

### Instances and Classes

In our study we will be dealing with entities of two very different kinds: classes and instances. Instances exist in the world but classes are created by humans. A soil order as it is defined in soil taxonomy (NRCS, 1999) is an example of a class of geographic objects, and a patch of soil with certain properties is an instance of that class. The formalism $\mathcal{I}(x, A)$ will stay for "$x$ is an instance of class $A$". Smith and Rosse (2003) suggest that a relation of instantiation has to be governed by two axioms: (1) $\mathcal{I}$ holds in every case between $x$ and $A$ and (2) an entity can be either a class or an instance, but not both.

### Relations between Instances

#### *Mereology*

The "part-of" relation is assumed to be primitive and is not defined further. $x < y$ will be used to say that $x$ is a proper part of $y$. Parthood relation can be axiomatized as (Simons, 2000; Varzi, 1996):

| | | |
|---|---|---|
| **PA1** | $\neg(x < x)$ | irreflexivity |
| **PA2** | $x < y \Rightarrow y \nless x$ | antisymmetry |
| **PA3** | $x < y \wedge z < x \Rightarrow z < y$ | transitivity |
| **PA4** | $\exists x (\phi(x)) \Rightarrow \exists z (\sum(\phi(x)))$ | summation principle |
| **PA5** | $x < y \Rightarrow \exists z (z < y \wedge z \neq x)$ | remainder principle |

The first two axioms (PA1-2) postulate that no entity can be a proper part of itself and an entity cannot be a part of its own part. Transitivity (PA3) does not always hold in the general case: a musician is a part of an orchestra, a hand is a part of musician, common sense says that a musician's hand is not a part of an orchestra (more discussion on this in Gerstl and Pribbenow, 1995). However we can safely assume PA3 to be true for the purpose of this study. Summation (PA4) ensures that arbitrary sums of entities exists and PA5 requires each entity to have at least two proper parts. As a result of the system of axioms PA1-5 entities satisfying "part-of" relationship always arrange themselves in trees (finite acyclic graphs) as shown on Figure 1(a). Configurations like Figure 1(b and c) are prohibited by PA1-5.

## Mereotopology

Mereotopology extends mereology by adding a connectedness relation. Connectedness is a primitive relation and is not defined further. Examples of connectedness relations are a hand that is connected to the body or a bay that is connected to the sea. To defined connectedness ($\mathcal{C}$) we will introduce a minimum set of axioms from (Cohn et al., 1997) that will satisfy the needs of this study:

| | | |
|---|---|---|
| **CA1** | $\forall x\,(\mathcal{C}\,(x,x))$ | reflexivity |
| **CA2** | $\mathcal{C}\,(x,y) \Rightarrow \mathcal{C}\,(y,x)$ | symmetry |
| **CA3** | $x < y \equiv \forall z\,(\mathcal{C}\,(z,x) \Rightarrow \mathcal{C}\,(z,y))$ | |

According to CA1 each instance is connected with itself and if $x$ is connected with $y$ then the reverse is also true (CA2). CA3 links connectedness with parthood.

## Relations between Classes

### *Granular partitions*

Organization of classes into hierarchical classification systems (taxonomies) is guided by the theory of granular partitions (Bittner and Smith, 2003). This theory has an advantage of being able to account for mereological structure of the objects and to handle temporal changes of class membership (gain and loss of members). Granular partition theory has two parts: (1) organization of partitions and relations between classes and (2) relations between partitions and objects of reality.

Partitions are instruments to subdivide reality into classes. We will use $\sqsubseteq_{\mathcal{P}}$ to designates subsumption (is-a) relation between two classes within a partition $\mathcal{P}$:

**GP1** $\quad A \sqsubseteq_{\mathcal{P}} B \equiv \forall x(\mathcal{I}(x,A) \Rightarrow \mathcal{I}(x,B))$

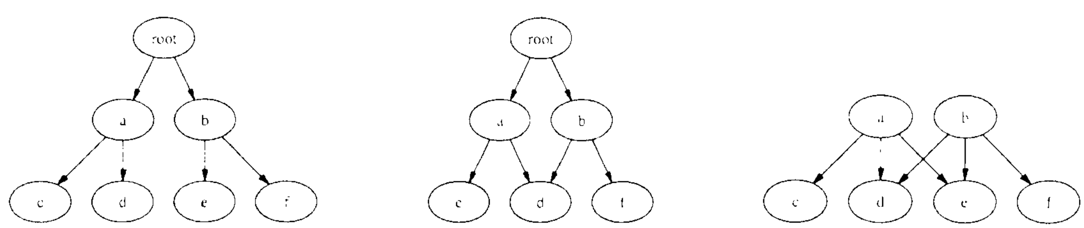
This way we say that if every instance of class $A$ is an instance of class $B$ then $A$ subsumes $B$. $\mathcal{P}$ will be omitted in most cases unless it causes confusion. The subsumption relation is reflexive, antisymmetric and transitive. Each partition must have a unique maximal class $A_{max}$:

**GP2** $\quad A_{max} \equiv \forall A\,(A \sqsubseteq A_{max})$

We also need to define minimal classes $A_{min}$:

**GP3** $\quad A_{min} \equiv \forall A\,(A \sqsubseteq A_{min} \Rightarrow A = A_{min})$

Each minimal class is connected to the maximal class through a finite chain of succeeding classes. We




(a) Proper tree (b) Overlaps (c) Multiple roots

Figure 1. Models of "part-of" relations.

have to ensure that there are no overlapping classes:

**GP4** $\exists A (A \sqsubseteq A_1 \wedge A \sqsubseteq A_2) \Rightarrow A_1 \sqsubseteq A_2 \vee A_1 \sqsupseteq A_2$

This axiom postulates that if two classes overlap then one always subsumes the other.

Partition is a very broad notion. Examples of partitions include: a list of the guests in the hotel, soil taxonomy, a biological species. Partitions may differ widely in their complexity. Simple partitions that consist only of a maximal class and minimal classes are called lists. Axioms GP1-4 ensure that classes in the partitions form proper trees. Thus Figure 1 (a, b and c) are also valid for partitions.

In the case where a partition projects on the entities that are involved in the part-of relation, we need another condition that we call *mereological monotony* (Bittner and Smith, 2003):

**GP5** $\mathcal{I}(x_1, A_1) \wedge \mathcal{I}(x_2, A_2) \wedge x_1 < x_2 \Rightarrow A_1 \sqsubseteq A_2$

GP5 requires a class to subsume another class if the instances that are instantiated by these classes are involved into the part-of relation. Mereological monotony ensures that partitions do not misrepresent mereological structure of the domains that they are projected on by explicitly prohibiting the case when an entity and its part are instances of the same class. Not all partitions satisfy this condition.

### Parthood for Classes

The "part-of" relationship described in the *Mereology* section applies for instances. In order to be able to operate with classes we need to define a parthood-like relationship for them. This relationship ($\lessdot$) is defined as follows (Smith and Rosse, 2003):

**CP1**

$$A \lessdot B \equiv \forall a(\mathcal{I}(a, A) \Rightarrow \exists b(\mathcal{I}(b, B) \wedge a < b)) \wedge \forall c(\mathcal{I}(c, B) \Rightarrow \exists d(\mathcal{I}(d, A) \wedge c < d))$$

We state that class $A$ is a part of class $B$ if its instances do not exist other than as parts of instance of class $B$ and instances of class $B$ do not exist except with instances of $A$ as its parts.

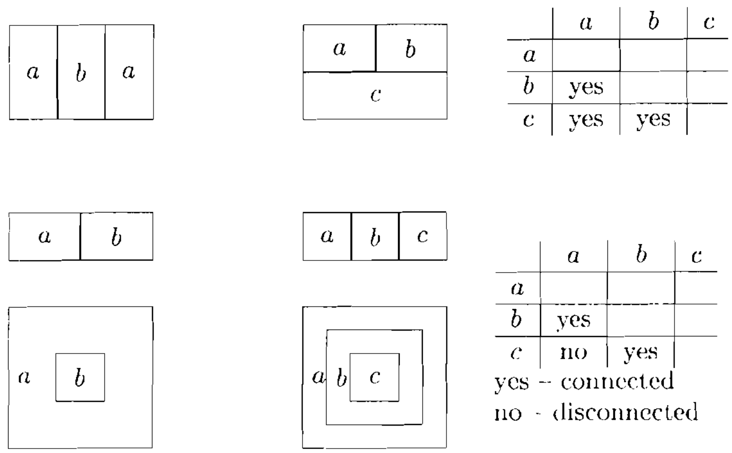
## FORMULATION OF THE CONSTRAINTS

First of all we have to define criteria for a spatial database to be suitable for application of mereotopological constraints. The first criterion is that the database must be topologically consistent. As it was shown in Bennett (1998) the arguments of relations PA1-5 and CA1-3 can be interpreted as non-empty open sets within an arbitrary topological space. This allows us to apply "part-of" and connectedness relations to the polygons in the two-dimensional vector topological model that represents a significant portion of existing geographic databases.

The next criterion imposes a restriction on the categories assigned to the polygons. The classes assigned to the polygons must belong to a partition that satisfies axioms GP1-5 and CP1. Many of the scientific taxonomies satisfy these conditions.

Finally, the database must contain some entities that can be described in terms of part-whole relationships and these entities should not be found in the partition that was used to classify the polygons. This information is not explicitly recorded in the database and expert knowledge is usually needed to determine its existence. These entities have to be defined as sets of polygon classes. Doing so creates a new partition that in the simple case could be just a list but it has to be created in a way to satisfy conditions GP1-5 and CP1. Instances of classes of the new partition will always be connected with each other due to CP1 and CA1-3. This connectedness information has to be tested against neighborhood information in the polygon topology.

In a simple case with only two instances (Figure 2(a)) the database has to be tested for the existence of instances of classes $A$ and $B$ that are not connected with each other. In cases involving three or more parts (Figure 2(b)) a connectedness matrix needs to be defined to allow or disallow boundaries between certain classes and this matrix has to be tested against polygon neighborhood information.




(a) two parts (b) three parts and connectedness matrices

Figure 2. Configurations of parts.

## CONCLUSIONS AND FUTURE WORK

Mereological (part-whole) relations are very often used to describe geographic objects and geographic concepts. Spatial databases typically contain large number of entities that are or can be involved in such relationships. Part-whole relationships have been very well studied and formalized in the ontological research. At the same time they are easy to understand intuitively. However, part-whole relations are seldom employed in the context of geographic information systems. This paper shows the possibility of using part-whole relations in order to define integrity constraints in spatial databases.

Future work will be concentrated in the direction of implementing and testing mereotopological constraints on a real-world dataset. Mereotopological constraints can be most effectively applied to voluminous spatial databases that contain information at various levels of generalization. At this point state- and county-level soil geographic databases are viewed as the best candidates for testing this theory.

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